# Approximate Representations of Random Intervals for Hybrid Uncertainty Quantification in Engineering Modeling

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## Abstract

Engineering modeling problems are frequently characterized by a large number of inputs with different forms and levels of uncertainty present on different collections of them. Propagating such hybrid uncertainties through high-complexity models (whether analytical or computational) is thereby especially challenging, as are elicitations and interpretations of both input and output uncertainties by domain experts and customers.

We have been developing an approach to the representation and propagation of hybrid uncertainties in engineering modeling applications based on quantities known as random intervals [Helton and Oberkampf 2004; Joslyn and Helton 2002; Joslyn and Kreinovich 2003]. These structures have a variety of mathematical descriptions, for example as interval-valued random variables, statistical collections of intervals, or Dempster-Shafer bodies of evidence on the Borel field [Joslyn and Booker, 2004].

One of the advantages of random interval structures is their ability to generalize more specific kinds of uncertainty quantities (for example, coarse-grained probability distributions, strong or weak statistical data, interval data, possibility distributions, and linguistic information represented as fuzzy sets) with a relative minimum of computational and mathematical complexity. Nonetheless, random intervals are not especially simple structures to represent or manipulate, and therefore methods which provide simpler, albeit approximate, representations of random intervals are highly desirable.

In this paper we report on a framework we are developing for this. In our approach, random interval quantities can be represented in increasingly simplified and approximate forms through p-box and trace structures respectively. A **p-box** [Ferson *et al.* 2002] is an ordered pair of monotonically increasing functions which together bound a collection of cumulative probability distribution functions. Each random interval necessarily generates a unique p-box, whose constituent CDFs are all of those consistent with the random interval. In turn, each p-box generates an equivalence class of random intervals consistent with it.

A **trace** [Joslyn 1997] is a fuzzy quantity on the line. Each p-box necessarily generates a unique trace, and stands as the fuzzy set representation of the p-box or random interval. Under different conditions it can take on the properties of a probability distribution,

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possibility distribution, or fuzzy interval quantity used in fuzzy arithmetic. In turn each trace generates an equivalence class of p-boxes.

The heart of our approach is to understand the tradeoffs between approximation and simplicity present when one or another of these structures (random intervals, p-boxes, or traces) are used for various operations. For example, Joslyn [2003] has argued that for elicitation and representation tasks, traces can be the most appropriate structure, and has proposed a method for the generation of canonical random intervals from elicited traces.

But alternatively, white-box models such as those built on algebraic equations propagate uncertainty through convolution operations on mathematical expressions of uncertainty-valued variables; in our case, random-interval-valued. But while convolution operations are defined on all three structures (random intervals, p-boxes, and traces), we have observed [Joslyn and Ferson 2004] that the results of only some of these operations are preserved as one moves through these three levels of specificity.

In this paper, we report on the status and progress of this modeling approach in the context of the issues concerning the relations between these mathematical structures.

**References**

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